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Subject: TOC

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4. TURING MACHINE

* Introduction to Turing Machine

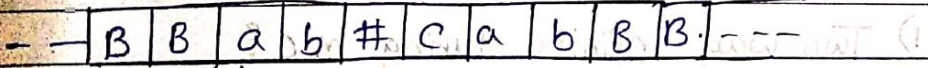
→ "Turing m/c is mathematical model which consist of
- an infinite length tape divided into cells on which
- input is given. It consist of head which reads the i/p tape."

- A state register stores the state of Turing m/c. After reading an i/p symbol, it is replaced with another symbol, if its internal state is changed and it moves from one cell to the right or left.
- If TM m/c reaches to final state, the i/p string is accepted otherwise rejected.

A TM m/c can be described as 7-tuple $(Q, X, \Sigma, \delta, q_0, B, F)$

- where, Q is finite set of state.
- X is tape alphabet
- Σ is i/p alphabet
- δ is transition function
- q_0 is initial state
- B is Blank symbol
- F is set of final state.

e.g.



↑ head

Example of Turing Machine.

- Turing m/c is more powerful than PDA.
- Turing m/c is capable of performing computation on i/p and producing a new result.

* Applications of Turing Machine :-

- 1) To read or write infinite tape.
- 2) to solve problem in computer science & testing limit of computation.
- 3) It is used to simulate other Turing machine.
- 4) Turing m/c is used to reverse string of any character.
- 5) it is used in algorithmic information theory.
- 6) Turing m/c is used for high performance computing, machine learning, SW engg and computer network.
- 7) Turing m/c is used to perform computation for computer system.
- 8) Turing m/c is used in theory of computation.

* Different ways of Extension of TM :-

→ In std TM, the tape is semi-infinite. It is bounded on the left and unbounded on the right side.

Some of the Extension of TM :-

- 1) Tape is of infinite length in both the direction.
- 2) Multiple heads and single tapes.
- 3) Multiple tape with each tape having its own independent head.
- 4) K-dimensional tape.
- 5) Non-deterministic Turing m/c.

1) Two-way infinite Turing m/c :-

→ In 2-way infinite TM, there is an infinite sequence of blank on each side of I/P string. In instantaneous description, these blocks are never shown.

2) A Turing m/c with multiple head :-

→ A TM with single tape can have multiple heads.

Let's consider TM with two heads H1 & H2.

Each head is capable of performing read/write operation.

B a a b a a b a B B

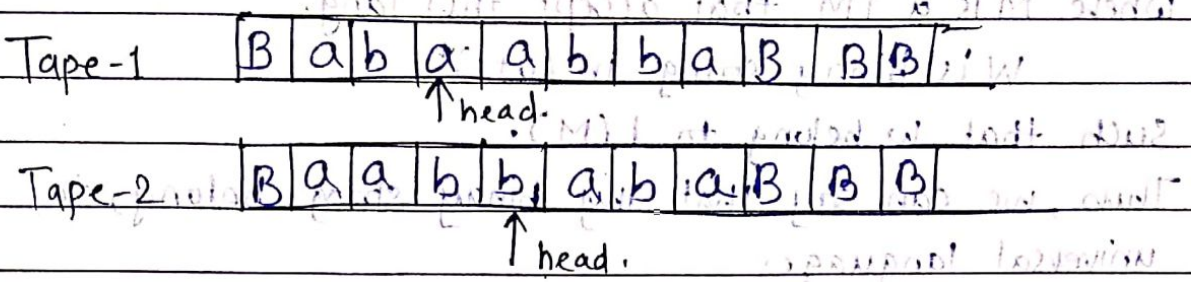
↑
H1

↑
H2

← TM with two head

3] Multi-tape Turing Machine:

- Multi-tape Turing machine has multiple tape tapes with each tape having its own independent head.
- Let's consider case of two tape Turing machine as shown in fig.



Two-tape Turing Machine

The transition behaviour of a two-tape Turing machine can be defined as given below as follows:

$$\delta(q_1, a_1, a_2) = \{(q_2, (S_1, M_1), (S_2, M_2))\}$$

where,

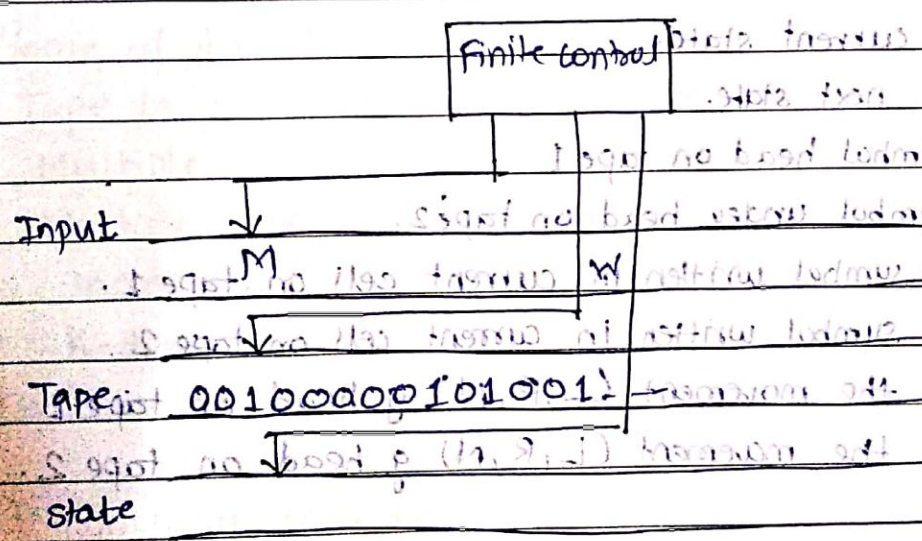
- q_1 is current state.
- q_2 is next state.
- a_1 symbol head on tape 1
- a_2 symbol under head on tape 2.
- S_1 is symbol written in current cell on tape 1.
- S_2 is symbol written in current cell on tape 2.
- M_1 is the movement (L, R, N) of head on tape 1.
- M_2 is the movement (L, R, N) of head on tape 2.

* Limitation of Turing Machine:

- 1) Computational Complexity theory: limitation is that they do not model strength of particular arrangement.
- 2) Concurrency: limitation is that they do not model concurrency well.
- 3) There are always halting concurrent systems with no I/P.
- 4) If two CFGs are given G_1 & G_2 then $L(G_1) \cap L(G_2) = \phi$ is undecidable.
- 5) Recursively Enumerable lang. and the halting problem.
- 6) TM machine is weak to describe the property the internet, evolution or robotics bcz it is closed model.

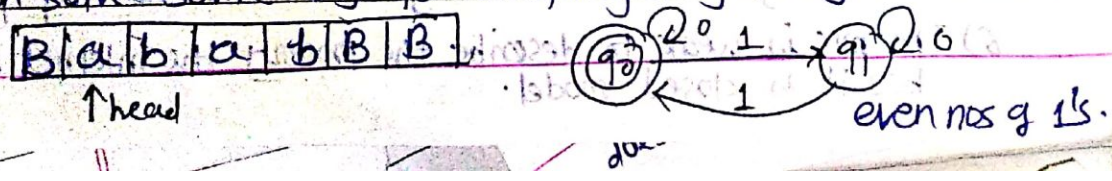
* Universal Turing Machine

- 1) The universal lang. is the set of binary strings which can be modeled by Turing m/c.
- 2) The universal lang. can be represented by pair (M, w) where M is a TM that accept this lang. w is binary string in $(0+1)^*$ such that w belong to $L(M)$. Thus, we can say that any binary string belongs to universal language.
- 3) The universal lang. can be represented by $L_u = L(U)$ where U is universal Turing m/c.
- 4) In fact, U is binary string. This binary string represent various codes of many Turing machine.
- 5) Thus, the universal Turing m/c is a Turing m/c which accepts many Turing m/c.



* Language acceptability by Turing machine

- TM accept all lang. even though there are recursively enumerable.
- Recursive means repeating the same set of rule for any no. of times.
- & enumerable means list of elements.
- TM also accepts Computable function, such as Addition, multiplication, subtraction, division, power fun, square fun & logarithmic function.
- we can solve some e.g. for accepting lang using TM.



* TM's Halting Problem :-

"Halting Problem is unsolvable"

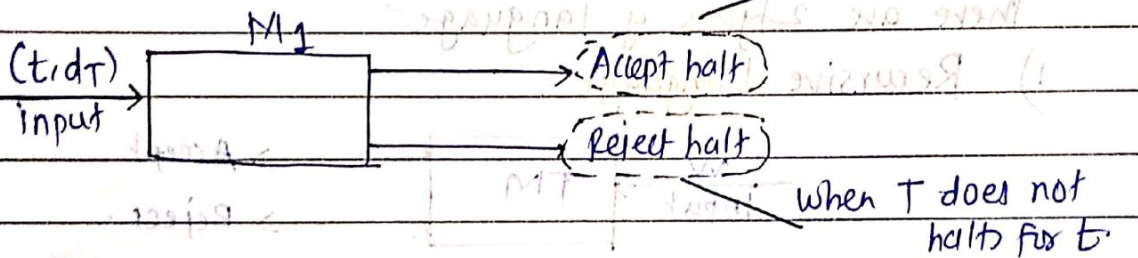
proof :-

Let's there exist, a TM M_1 which decides whether or not any computation by a TM T will ever halt when a description d_T of T and tape t of T is given.

Then for every i/p (t, d_T) to M_1 if T halt for i/p t .

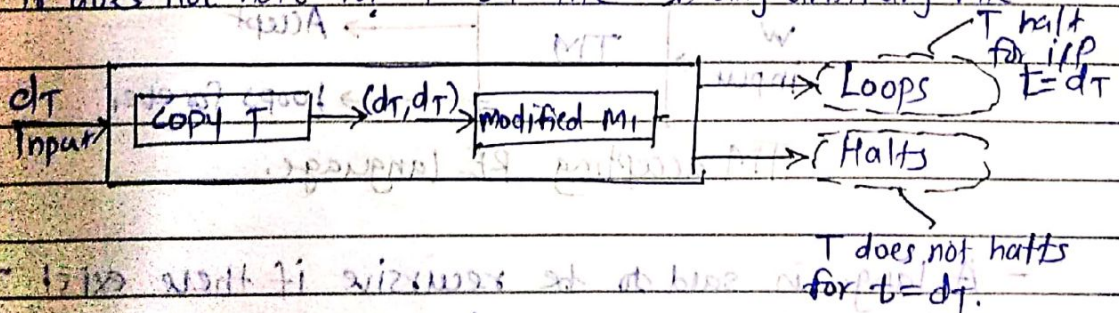
M_1 also halts which is called accept halt.

- similarly if T does not halt for i/p t then the M_1 will halt which is called reject state.



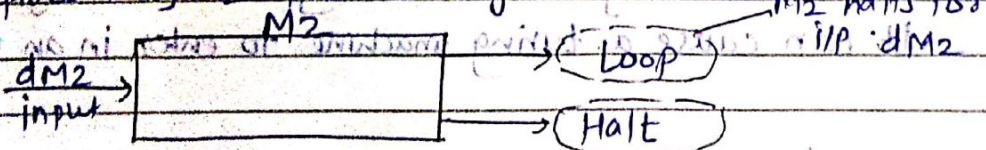
When we will consider another TM M_2 which takes i/p d_T .

First copy d_T and duplicate d_T on its tape and then this duplicate tape info is given as i/p to M_1 . But M_1 is modified with the modification that whenever M_1 is supposed to reach an accept halt, M_2 loop forever. Hence behaviour of M_2 is given. It loops if T halts for i/p $t = d_T$ and halts if it does not halt for $T = d_T$. The T is any arbitrary m/c.



As M_1 itself is one TM, we will take $M_2 = T$, that means

we will replace T by M_2 from above given m/c.



This is contradiction, that means M_1 which can tell whether any other m/c TM will halt on particular i/p does not exist. Hence halting problem is unsolvable.

M_2 does not halts for i/p d_{M_2} .

* TM and Type 0 Grammars.

→ TM can accept Type 0 grammar.

- The type 0 grammar is said to be unrestricted grammar.

e.g. restricted lang. are almost all natural lang.

- The class of lang. accepted by type 0 grammars are called recursively enumerable language.

- The prodⁿ can be in form $\alpha \rightarrow \beta$

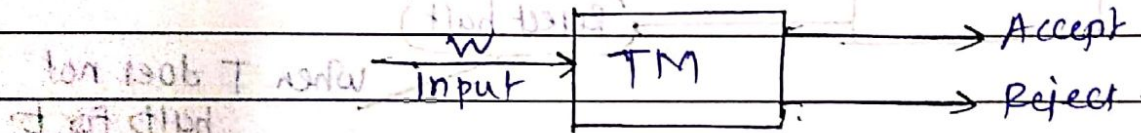
where α is string of terminal and non-terminal.

|| with atleast one non-terminal and α can't be null.

β is string of terminal & non-terminal.

There are 2 types of language:

1) Recursive Language:



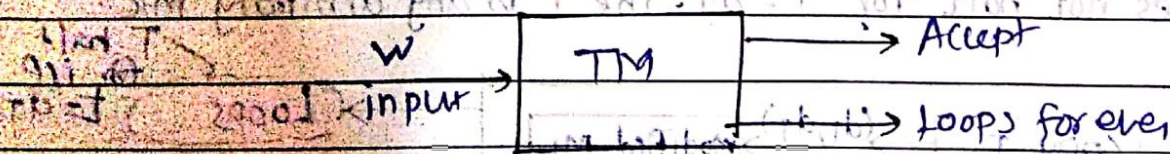
TM Accepting Recursive Language

A lang. is said to be recursive if there exists a TM.

that accept every string in lang. and every string is

rejected if it is not belonging to that lang.

2) Recursively Enumerable Language:



TM accepting RE language.

- A lang. is said to be recursive if there exist TM

that accept every string belonging to that language.

And if the string does not belong to that language then

it can cause a Turing machine to enter in an infinite loop.