UNIT - II

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REGULAR EXPRESSION (RE)

* Regular Expression/set :-

→ "The set of string accepted by finite automata is known as Regular Language."

operator :- This language can also be described in a compact form using a set of operators.

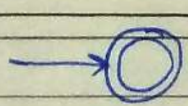
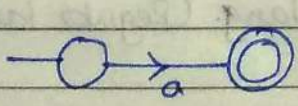
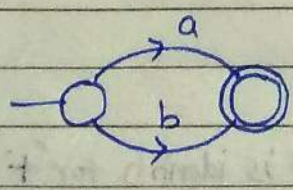
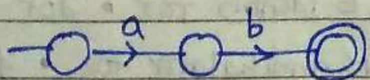
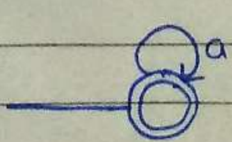
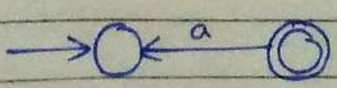
These operators are :-

1) + , union operator
2) • , Concatenation operator
3) * , star / closure operator.

"Regular Expression :-

Def :- An expression written using set of operators (+, •, *) and describing a regular language is known as Regular Expression.

The Regular expression for some basic Automata.

Automata	Language	Regular expression
	{ϵ}	R.E = ϵ
	{a}	R.E. = a
	{a, b}	R.E. = a+b.
	{ab}	R.E = a•b.
	{ϵ, a, aa, aaa...}	R.E = a^*
	ϕ	R.E = ϕ.

* Precedence of Operators :

→ operator are associated with operand in particular order. The Precedence of operators for regular expression is as follow :

1) The star operator has the highest Precedence
2) The concatenation or "dot" operator comes next in precedence
3) Union or "+" operators comes last in the Precedence.

* Algebraic laws for Regular Expression :

→ There are number of laws for algebraic laws :

1) Associativity and commutativity.
2) Identities and annihilators.
3) The idempotent law.
4) laws of involving closures. 5) Distributive law.

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1) Associativity & Commutativity

- Commutative laws for union says that the union of two regular languages can be taken in any order.
- For any two lang. L and M.

$$L+M = M+L$$

- The Associative law holds for union of two lang. (Regular lang)

$$(L+M)+N = L+(M+N)$$

2) Identities and annihilator :

ϵ is identity and ϕ is annihilator

There are 3 law

1) $\phi + L = L + \phi = L$ ϕ is identity for '+'
2) $\epsilon \cdot L = L \cdot \epsilon = L$ ϵ identity for $\cdot$ dot
3) $\phi \cdot L = L \cdot \phi = \phi$ is annihalator for $\cdot$ dot

3) Distributive law :

1) Left distributive law of concatenation over union

$$L(M+N) = LM+LN.$$

2) Right distributive law of concatenation

$$(M+N)L = ML+NL$$

4. Idempotent Law :-

It says that the union of two identical expression can be replaced by one copy of expression.

$$L + L = L$$

5. Laws of Involving closures.

These laws include :-

1. $(L^*)^* = L$
2. $\phi^* = \epsilon$
3. $\epsilon^* = \epsilon$
4. $L^* = LL^* = L^*L$
5. $L^* = L^* + \epsilon$

* Pumping Lemma for Regular Language :-

Def :- "Let L be a regular language and $M = \{Q, \Sigma, \delta, q_0, F\}$ be a finite automata with n-states. Language L is accepted by M. Let $w \in L$ and $|w| \geq n$ then w can be written as xyz where.

i) $|y| > 0$ ii) $|xy| \leq n$

iii) $xy^iz \in L$ for all $i \geq 0$ here y^i denotes that y is repeated or pumped i times"

- pumping lemma should be used to establish that a given language is not regular

* Closure properties of Regular Language :-

"If an operation on regular lang. generates a Regular lang. then we say that "the class of regular lang. is closed under above operation"."

Some of the closure properties :-

1) Union
2) Difference
3) Concatenation
4) Intersection
5) Complementation
6) Kleene star.

* Regular Language is ~~under~~ closed under union :-

→ Let $M_1 = \{S, \Sigma, \delta_1, S_0, F)$ and
$M_2 = (Q, \Sigma, \delta_2, q_0, G)$ be two given automata.

To prove closure property, we need M_3 variable which accept every string accepted either by M_1 or M_2.

$$\therefore L(M_3) = L(M_1) \cup (M_2)$$

* Regular lang. is closed under Concatenation :-

→

$$L(M_3) = L(M_1) \cdot L(M_2)$$

* Regular lang is closed under Kleene star (closure)

→

$$L(M_2) = L(M_1)^*$$

* Difference :- $L(M_3) = L(M_1) - L(M_2)$

* Intersection :- $L(M_3) = L(M_1) \cap L(M_2)$

* Complementation :- $L(M_1) = \overline{L(M_1)}$

* Reversal :- $L = \{aab, abb, aaa)$
then $L^R = \{baa, bba, aaa\}$;

* Show that the set $L = \{ b^{i^2} \mid i \geq 1 \}$ is not regular.

→ We have to prove that the lang $L = b^{i^2}$ is not regular.
This lang is such that nos of b's is alway a perfect square.
For example,

if we take $i = 1$

$L = b^{1^2} = b$ then length $= 1^2$

$= b^{2^2} = bbbb$ then length $= 2^2$

and so on.

Now let us consider, $L = b^{n^2}$ where length n^2
it is denoted by Z

$|Z| = n^2$

By pumping lemma,

$Z = uvw$ where $1 \leq |v| \leq n$.

As $Z = uv^iw$ where $i = 1$.

Now we will pump v i.e. $i = 2$

As we made $i = 2$ we have added one n^2 $1 \leq |v| \leq 2$

$n^2 + 1 \leq |uvw| \leq n + n^2$

Thus, string lies bet^n 2 consecutive Perfect square.
But string is not perfect square, Hence we can say the given lang. is not regular.

For example :-

$L = b^{i^2}$

$i = 2$

$L = bbbb$

$L = uvw$

Assume $uvw = bbbb$

$u = b$

$v = bb$

$w = b$.

By pumping lemma, even we pump v i.e increase v. then lang. should show the length as perfect square.

$= uvw$

$= uv \cdot v \cdot w$ $\because$ $u = b$, $v = bb$, $v = bb$, $w = b$

$= bbbbbb$.

The length of b is not Perfect square. so it is not regular.

* Prove / Disprove that the language L given by
$L = \{ a^m \cancel{b} a^n \mid m \neq n$ and n are +ve integer$\}$ is regular.

→ Let $L = \{a^m a^n \mid m \neq n\}$ be a language
Assume that L is regular lang., Now consider case
i) Case 1 : $Z = aaabbbb \in L$.
By pumping lemma if $Z = uvw$ then we pump some string to make $Z = uv^iw$ then ~~if z~~.
if $Z \in L$ then such lang. is called regular.
Let,

$$Z = \underbrace{a}_{u}\,\underbrace{a\,a\,b\,b}_{v}\,\underbrace{b\,b}_{w}.$$

if $z = uv^iw$, and if $i = 2$ then $Z = uvvw$.

$$z = \underbrace{a}_{u}\,\underbrace{aabb}_{v}\,\underbrace{aabb}_{v}\,\underbrace{bb}_{w}$$

$$z = a^3 b^2 a^2 b^4 \in a^m b^n \in L.$$

ii) case 2 : Assume $L \in Z$ such that

$$Z = \underbrace{a}_{u}\,\underbrace{aa}_{v}\,\underbrace{a\,bbbbbb}_{w}$$

By pumping lemma,
$Z = uv^iw \in L$ for regular lang. If $i = 2$
$Z = uvvw$

$$z = \underbrace{a}_{u}\,\underbrace{aa}_{v}\,\underbrace{aa}_{v}\,\underbrace{abbbbbb}_{w}$$

$$z = a^6 b^6 \in a^m a^n \text{ because } m = n.$$

From these case, we get $Z \in L$, Thus our assumption of L being regular is wrong.
Hence, given language $\{L = a^m b^n \mid m \neq n\}$
is not regular // Disproved