

Context Free Grammars & Language

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* Context Free Grammar (CFG) $\stackrel{\circ}{=}$ (Regular Grammar)

"A context free Grammar G is a quadruple (V, T, P, S)

Where, V is set of variables

T is set of terminals.

P is set of productions

S is a special variable called start symbol $S \in V$.

A production is of the form

$V_i \rightarrow \alpha_i$ where $V_i \in V$ and α_i is string of terminal and variables.

Notations :-

1. terminal are denoted by lower case letters a, b, c or digits $0, 1, 2, \dots, 9$.

2. Non-terminal (variables) are denoted by capital letter A, B, C, \dots, Z .

3. A string of terminals or a word $w \in L$ is represented using u, v, w, x, y, z .

4. Sentential form is a string of terminals and variables and it is denoted by α, β, γ etc.

e.g. Anand writes, Anand reads, Sunny watch

A sentence for above word can be written as

$\langle \text{sentence} \rangle \rightarrow \langle \text{noun} \rangle \langle \text{verb} \rangle$

Noun \rightarrow Anand | Sunny

Verb \rightarrow writes | reads | watch.

$$P = \left\{ \begin{array}{l} \text{sentence} \rightarrow \text{noun} | \text{verb} \\ \text{noun} \rightarrow \text{Anand} | \text{Sunny} \\ \text{verb} \rightarrow \text{writes} | \text{reads} | \text{watch} \end{array} \right\}$$

* Sentential form α

In Sentential form, derivation starts from the start symbol through finite application of productions.

A string α is derived so far consist of terminals and non-terminals:

$$S \xrightarrow[G]{*} \alpha \mid \alpha \in (V \cup T)^*$$

- A final string consist of terminals.
- In left sentential form, leftmost symbol is picked up for expansion.
- In right sentential form, rightmost symbol is picked up for expansion.
- A string can be derived in two ways:
 - 1) Leftmost Derivation
 - 2) Rightmost Derivation.

e.g.

$$\begin{aligned} S &\rightarrow A1B \\ A &\rightarrow 0A \mid \epsilon \\ B &\rightarrow 0B \mid 1B \mid \epsilon \end{aligned}$$

where G is given by (V, T, P, S)

with

$$V = \{S, A, B\}$$

$$T = \{0, 1\}$$

$$P = \left\{ \begin{array}{l} \text{production } S \rightarrow A1B \\ A \rightarrow 0A \mid \epsilon \\ B \rightarrow 0B \mid 1B \mid \epsilon \end{array} \right\}$$

$$S = \text{start symbol.}$$

"A language of grammar can be defined by creating the production rules for the given condition.

The language generated by context free grammar is called as Context free Language (CFL)

* Ambiguous Grammar

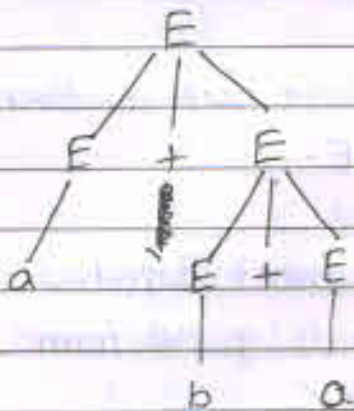
A grammar is said to be ambiguous if the language generated by grammar contains some string that has two different parse tree.

e.g.

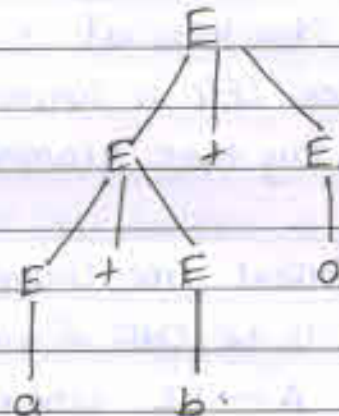
$$E \rightarrow E + E \mid a \mid b ;$$

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A string $a + b + a$ is generated by given grammar.



Parse tree 1



Parse tree

As above, two parse tree are generated for string $a + b + a$ so that's why it is ambiguous grammar.

* Elimination of Useless Symbol & Production

A grammar may contain symbols & productions which are not useful for derivation of string.

Two types of symbol are useless

1. Non-generating symbols
2. Non-reachable symbols

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* Normal form :-

There are 2 imp normal form

- 1) Chomsky's Normal form
- 2) Greibach Normal form

1) Chomsky's Normal form (CNF) :-

→ The CNF can be defined as

Non-terminal → Non-terminal • Non terminal.

Non-terminal → Terminal

The given CFG is converted in the above format then we can say that grammar is in CNF.

“ A Context Free Grammar (CFG) without ϵ -production is said to be CNF if every production is of the form:

1. $A \rightarrow BC$ where $A, B, C \in V$
2. $A \rightarrow a$ where $A \in V$ and $a \in T$.

The grammar should have no useless symbols.

Every CFG without ϵ Productions can be converted into an equivalent CNF form.”

* Algorithm for CFG to CNF.

→ 1) Eliminate ϵ -production, unit production & useless symbol from grammar.

2) Every variable deriving a string of length 2 or more should consist only of variables.

⊙ $A \rightarrow \alpha$ with $|\alpha|$ should consist on variables.

e.g. $A \rightarrow V_1 V_2 a V_3 b V_4$

as $A \rightarrow V_1 V_2 C_a V_3 C_b V_4$ and adding two prod. :-

$C_a \rightarrow a, C_b \rightarrow b$

3) Every prod., deriving 3 or more variables

($A \rightarrow \alpha$ with $|\alpha| \geq 3$) can be split into cascade production with each deriving a string of two variables.

e.g. $A \rightarrow X_1 X_2 X_3 \dots X_n$ where $n \geq 3$ with $|\alpha| \geq 3$

e.g. consider production $A \rightarrow X_1 X_2 \dots X_n$ where $n \geq 3$ and

X_i are variable. $A \rightarrow X_1 C_1$
 $C_1 \rightarrow X_2 C_2$
 $C_2 \rightarrow X_3 C_3$

* Greibach Normal Form (GNF)

→ "A context free Grammar $G = (V, T, P, S)$ is said to be in GNF if every production is of the form:

$$A \rightarrow a\alpha$$

where, $a \in T$ is a terminal &

α is string of zero or more variable

- The language $L(G)$ should be without ϵ
- RHS of each prod should start with a terminal followed by a string.

- Removing Left Recursion:

→ Elimination of left recursion is an important step in algo. used in conversion of a CFG into GNF form.

- Left Recursive grammar:

A prod of the form $A \rightarrow A\alpha$ is called left recursive as the left hand side variable appears as the first symbol on the right-hand.