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INFORMATION TECHNOLOGY.

TOC INSEM 2022 SOLUTION

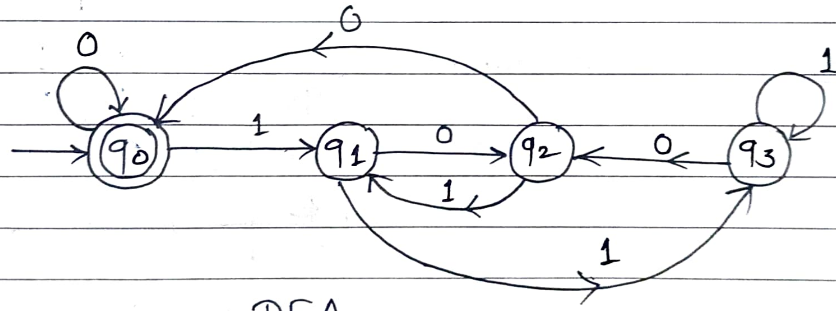
Q. 1 a) Design DFA which accepts Binary nos divisible by 4.

→ 1) Logic ÷ - Input are Binary number $\Sigma = \{0, 1\}$
- it should be divisible of 4

e.g. 00, 100, 1000, 1100, 10000 ... so on.

Remainder		Input.
q_0	0	00, 100, 1000, 1100, 10000 ...
q_1	1	01, 101, 1001, 1101 ...
q_2	2	10, 110, 1010 ...
q_3	3	11, 111, 1011, 1111 ...

2) State Transition Diagram ÷



DFA

3) State Transition Table ÷

	Input	
states	0	1
→ q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3

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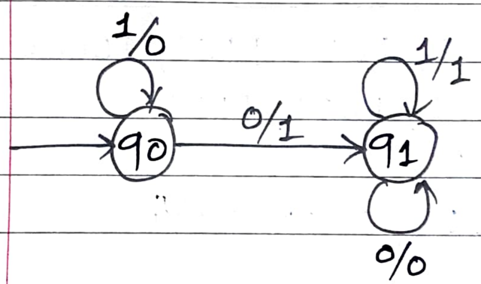
Q 1 b) Design Mealy Machine to increment Binary number by 1. Write down transition table.

→ 1) Logic ÷

- We can read Binary Number from LSB one bit at a time
- We can replace each 1 by 0 until we get 1^{st} 0.
- As we get 1^{st} 0 we will replace it by 1. then keep remaining bit as it is.

2) State Transition Diagram ÷



Mealy Machine

e.g.
0000 → 0001
0001 → 0010
0010 → 0011
0011 → 0100
0100 → 0101
0101 → 0110
0110 → 0111 so on

0 1 1 0
as it is ↓ change
0 1 1 1

3) State Transition table

	Next state			
state	0	o/p	1	o/p
q_0	q_1	1	q_0	0
q_1	q_1	0	q_1	1

Q.1.c] Convert the foll. NFA − ∈ to DFA.

states / Input	δ: ∈	a	b	c
→ p	{p}	{q}	φ	φ
q	{r}	φ	{q}	φ
r*	φ	φ	φ	{r}

→ Step 1 ÷ Find ∈-closure of all states

p = {p, q, r}
q = {q, r}
r = {r}

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Q.1 c) NFA-ε to DFA :-

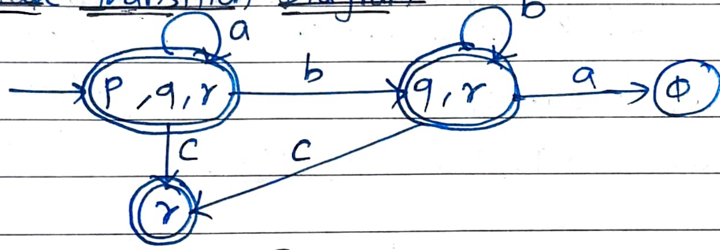
→ Step-2 :-

Take P as start state But In ε-NFA, we have to consider it's ε-closure of P. or any new states.

ε-closure new state	a	b	c	
p, q, r.	p, q, r.	q, r.	r	q, r is new state
q, r	φ	q, r.	r	No new state
r*	φ	φ	r	No new state.

Step 3:

State transition Diagram



Final DFA.

OR

Q.2 a) Define following term with Proper Examples :-

i) Alphabets :-

An alphabet is a finite, non-empty set of symbol. Alphabets is denoted by Σ this symbol.

e.g. Σ = {0, 1} , Σ = {a, b}

ii) String :-

A string is a finite sequence of symbols from alphabet.

e.g. 1010 is string from Binary alphabets.

2345 is string from Decimal alphabets.

iii) Natural language :-

- Natural language are the language that people speaks likes Marathi, Hindi, English etc.

- These languages are not designed by people.

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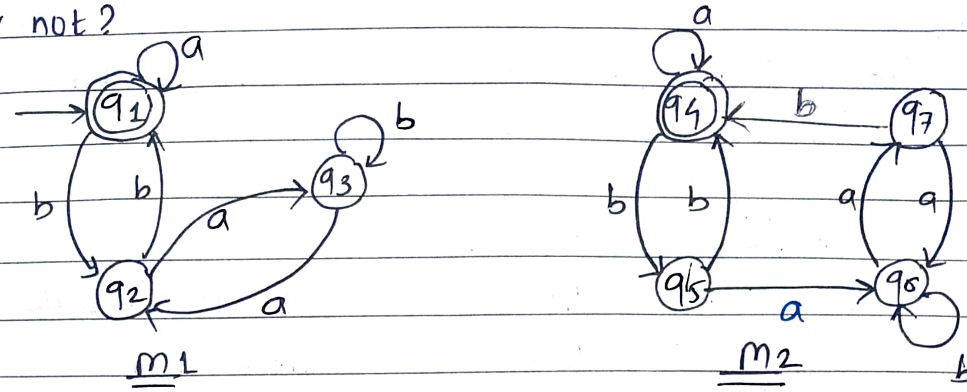
Q.2 b) Show whether the foll. automata M1 & M2 are equivalent or not?



→ Step-1 :

Consider start state of M1 & M2 to get new state from M1 & M2. ~~Here~~,

If new state found means equivalent (For all states) [both new states should final or Non-final states)

$\delta(\{q_1, q_4\}, a) = \{q_1, q_4\}$
$\delta(\{q_1, q_4\}, b) = \{q_2, q_5\}$ → new state

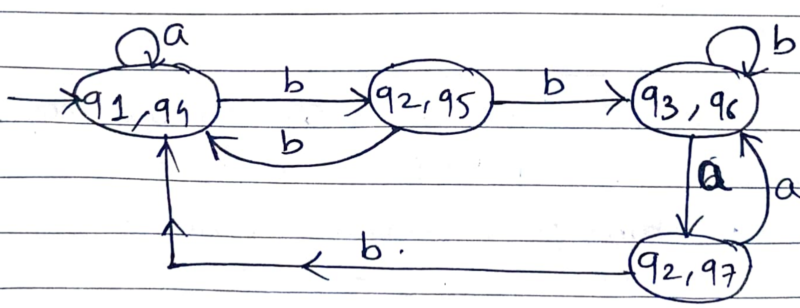
} Check outgoing edges from states.

Step-2 : $\delta(\{q_2, q_5\}, b) = \{q_1, q_4\}$
$\delta(\{q_2, q_5\}, a) = \{q_3, q_6\}$ → new state

Step 3 : $\delta(\{q_3, q_6\}, b) = \{q_3, q_6\}$
$\delta(\{q_3, q_6\}, a) = \{q_2, q_7\}$ → new state

$\delta(\{q_2, q_7\}, a) = \{q_3, q_6\}$
$\delta(\{q_2, q_7\}, b) = \{q_1, q_4\}$

} No new states. so stop.



M3

Every state in M3 has been expanded.
Hence, Construction of DFA is over. 4 states are generated

1) (q_1, q_4) → q_1 & q_4 Both are final states
2) (q_2, q_5) → q_2 & q_5 both are non-final states
3) (q_3, q_6) → Non final states
4) (q_2, q_7) → Non-final states, Thus, M1 & M2 are equivalent.

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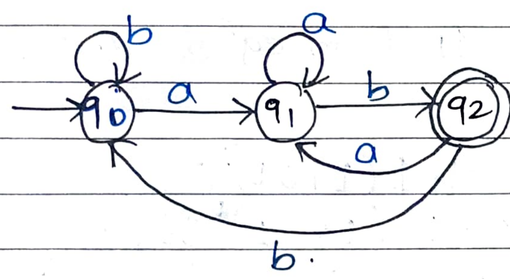
Q.2 c) Construct DFA over the alphabets {a, b} for accepting the strings ending with "ab"

→ 1) Logic :-

- Input are a, b $\Sigma = \{a, b\}$
- string ends with ab.
- e.g. ab, aab, bab, bbab --- so on

2) State Transition Diagram :-

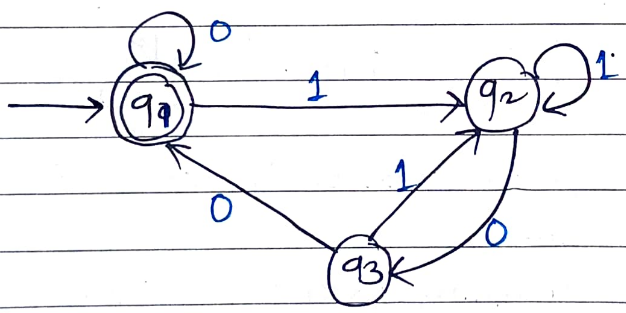


DFA

3) State Transition Table

States	Inputs a	b
→ q_0	q_1	q_0
q_1	q_1	q_2
* q_2	q_1	q_0

Q.3 a) Find the Regular Expression for set of strings recognized by the given FA using Arden's theorem.



→ step1 :- Find equations for all states. & simplify if possible.

$$q_1 = q_1 0 + q_3 0 + \epsilon \quad \text{—— (1)}$$
$$q_2 = q_1 1 + q_2 1 + q_3 1 \quad \text{—— (2)}$$
$$q_3 = q_2 0 \quad \text{—— (3)}$$

Substitute the value of q_3 in eq^n (1) & (2),

We get, $q_1 = q_1 0 + q_2 00 + \epsilon$ —— (4)

$$q_2 = q_1 1 + q_2 1 + q_2 01$$
$$q_2 = q_1 1 + q_2 (1 + 01) \quad \text{—— (5)}$$

From Arden's Theorem

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Q. 3 a) Arden's Theorem :-

→ From Arden's Theorem

$$q_2 = \underbrace{q_1 1}_{Q} + \underbrace{q_2}_{R}\underbrace{(1+01)}_{P} \quad \text{--- } ⑤$$

$$R = Q \quad R \quad P$$

So, We can Rewrite

$$R = QP^*$$

$$\boxed{q_2 = q_1 1(1+01)^*} \text{ --- } ⑥$$

Substitute value of q_1 from eqⁿ ⑥ in eqⁿ ④,

We get,

$$q_1 = q_1 0 + q_1 1(1+01)^* 00 + \epsilon$$

$$\underbrace{q_1}_{R} = \underbrace{q_1}_{R}\underbrace{[(0+1)(1+01)^* 00]}_{P} + \underbrace{\epsilon}_{Q}$$

By Arden's Theorem.

$$R = RP + Q$$

We can Rewrite it as

$$R = QP^*$$

$$q_1 = \epsilon \cdot [(0+1)(1+01)^* 00]^*$$

$$\boxed{R.E = [(0+1)(1+01)^* 00]^*}$$

b) Determine the Regular Expression over $\Sigma = \{0,1\}$ for the following:

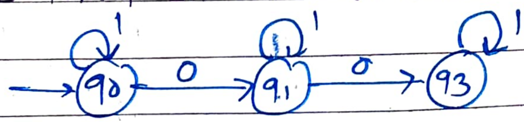
i) All the string Containing exactly two 0's.

ii) All the string do not end with 01

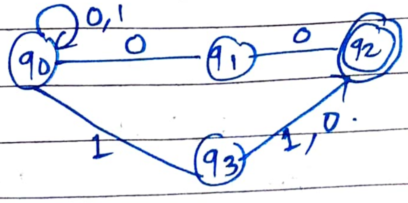
iii) All the string Containing 1 as third character from end.

→

i) $1^* 0 1^* 0 1^*$

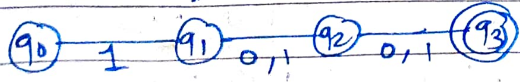


2) $(0+1)^* + (00 + 11 + 10)$



3) $1 \cdot (0+1) \cdot (0+1)$

↓



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Q.3 c) Explain the following term

i) Kleene closure

ii) Positive closure

→ 1) Kleene closure :

Given an Alphabet Σ the Kleene closure of Σ is a lang given by

$$\Sigma^* = \underbrace{\Sigma^0}_{\epsilon} \cup \underbrace{\Sigma^1}_{\text{string of length 1}} \cup \underbrace{\Sigma^2}_{\text{string of length 2}} \ldots$$

For Example :

If $\Sigma = \{x\}$, then

$\Sigma^* = \{\epsilon, x, xx, xxx \ldots\}$

If $\Sigma = \{0,1\}$ then

$\Sigma^* = \{\epsilon, 0, 1, 10, 01, 11 \ldots\}$

ii) Positive Closure :

L^+ is a Positive closure of language.

L^+ is the set of all finite string formed by Concatenating word from L.

Any word can be used one / more time.

Example :

If $L = \{a\}$ then

$L^+ = \{a, aa, aaa \ldots\}$

If $L = \{aa, b\}$ then

$L^+ = \{b, aa, bb, bbb, baa \ldots\}$

In this ϵ - (epsilon) input are not allowed)

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Q.4 a) Explain Any three closure Properties of Regular language.

→ 1) Union of Two Regular languages is regular

2) The Intersection two Regular language is regular

3) The closure operation on a Regular language is regular.

4) The Concatenation of Regular language is regular.

5) The Concatenation of Regular language is Regular

1) Union of Two Regular language is Regular.

→ If L1 & L2 are Regular. then

They have Regular Expression $L1 = L(R_1)$ and
$L2 = L(R_2)$

Then, $L1 \cup L2 = L(R_1 + R_2)$

Thus, We get $L1 \cup L2$ as Regular language.

(Any lang, given by Some Regular Expression is Regular)

2) The closure Operation on Regular language is Regular

→ - If Language L is regular. then it can be expressed as
$L1 = L(R_1^*)$

Thus, for closure operation a lang. can be expressed as a lang. of Regular Expression.

Hence, L is said to be Regular language.

e.g.

$a^* = \{\epsilon, a, aa, aaa \ldots\}$.

3) The Concatenation of Regular Language is Regular.

→ If L1 & L2 are two language. then $L1 \cdot L2$ is regular. In other word Regular language is closed Under Concatenation.

If L1 & L2 are regular then they can be expressed as $L1 = L(R_1)$ and $L2 = L(R_2)$

Then $L1 \cdot L2 = L(R_1 \cdot R_2)$

Thus, we get a Regular language.

Hence, it is proved that Regular lang. is closed Under Concatenation.

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Q.4 b) What is Regular Expression? Explain in brief the application of Regular Expression?

→ **Regular Expression:**

"The language accepted by Finite Automata can be easily described by Simple expression is called as Regular Expression."

Example:

$$R.E. = a^*$$

$$R.E = (a+b)^* \text{ OR } (0+1)^*$$

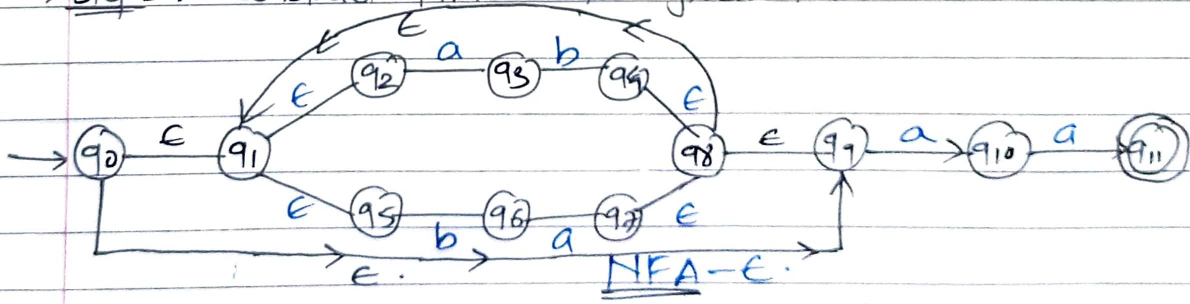
- **Application of Regular Expression:**

1) Regular expression are useful in wide variety of text Processing tasks and more generally word Processing.
2) Common application includes
   - data validation, data wrangling, Simple Parsing etc.
3) It is also useful in Internet Search Engine
4) Verify the structure of string
5) Search / Replace / rearrange part of string.
6) Split a string into tokens

c) Construct a NFA for foll. R.E. using Direct Method:

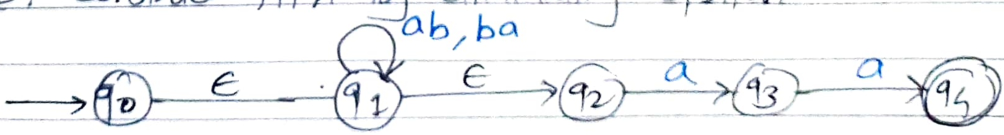
$$R.E. = (ab+ba)^* aa$$

→ **Step 1:** Construct NFA-ε for given R.E.

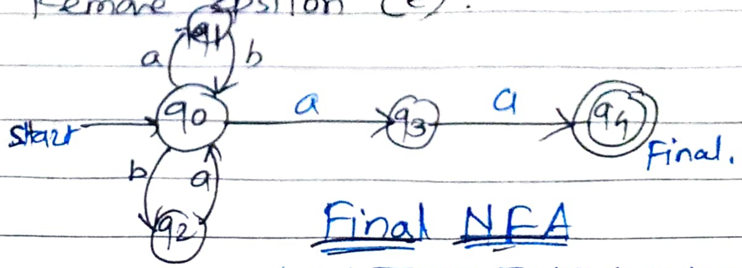


NFA-ε.

Step 2: Construct NFA By eliminating Epsilon.



Step 3: Remove Epsilon (ε).



Final NFA

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THE END